

Utility-Based Valuation of Risk

Introduction

While it seems intuitive that risky assets should be worth less than risk-free assets, this observation does not lead to specific prices for risky assets. Specific prices are essential for people putting real money into actual assets.

The present model derives specific prices for risky assets by analyzing the behavior of agents with curved utility functions of the form $U = f(X)$, where X is in dollars and U is utility. The application here considers the square root utility because that function is computationally convenient and handles the case of $X = 0$. Another common case is $U = \log(X)$.

The Model

An agent with square root utility ($U = \sqrt{\$}$) faces a coin toss that is worth 100 with probability 0.50 and 4 with probability 0.50. The expected dollar value of this asset is $\$52 = 0.50(100) + 0.50(4)$.

Exercises

1. Show that the value of this asset to the agent is only \$36.
2. What is the value of the asset if the probability of 100 is 0.90 and the probability of 4 is 0.10?
3. The difference between the expected dollar value and the value to the agent is known as the value of the risk. What is the value of the risk in the two cases above?
4. Why is the value of the risk different in the two cases?